Investigating Effective Remediation of Integer Operations at the High School Level

By

Sarah (Hagan) Carter

A Research Project
Submitted to the College of Education
Graduate Programs
University of Texas Arlington

In partial fulfillment of requirements for the degree of
Master of Education
2016

MAJOR: Curriculum and Instruction

Math Education

Abstract

The purpose of this study was to compare the use of the number line model and the use of memorized rules to determine if one method is more effective in terms of increases in fluency and retention than the other in a remediation setting. Additionally, the study compared the results of remediating Algebra 1 and Algebra 2 students to determine if the number line model was more effective with students enrolled in upper-level classes. The study consisted of 1 teacher-researcher, 49 Algebra 1 students, and 23 Algebra 2 students at a small, rural, public high school in the south-central United States. Students were given a pre-test and randomly placed in two groups. The students in the treatment group received remediation which focused on using a number line model to solve integer problems, while the students in the control group received remediation focusing on memorized rules. Over the course of three weeks, students worked five minutes per day on improving their fluency with integer operations using flash cards and their assigned method. Students were tested at the end of the three weeks, then again five months later. The study found that the use of the two methods were equally effective in improving students' fluency with integer operations. The study also found that the differences in retention levels between the students in the two groups were not statistically significant; therefore, the two methods are equally effective at ensuring retention of the remediated material. Finally, the study found that the use of the number line model for remediating integer operations is equally effective with students enrolled in upper-level and lower-level math classes.

Table of Contents

Table of Tables

Table of Figures

Chapter 1

Introduction

Mathematics is an area of struggle for many American students. The National Mathematics Advisory Panel (NMAP) (2008) reports American students are falling behind students of many other countries in the area of mathematics. Of the nations that participate in PISA, the United States ranks 32nd in mathematical proficiency. Only 32% of the tested members of the Class of 2011 earned a score of proficient; comparatively, 75% of students in Shanghai and 58% of students in Korea scored proficient. Twenty-two separate countries significantly outperformed the United States in the percentage of students scoring proficient (Peterson, Woessmann, Hanushek, and Lastra-Anadon, 2011).

Efforts to improve mathematics education have produced results. The National Assessment of Educational Progress (NAEP) began measuring the math ability of American students in 1973. Results from 2012 show 9-year-olds and 13-year-olds are scoring higher now than in the early 1970s, but the average math score for 17-year-olds is virtually unchanged between 1973 and 2012 (NCES, 2013a). Though scores are improving for some age groups, American students continue to have room for improvement. NAEP data from 2012 shows only 47% of 9-year-olds demonstrated an understanding of the four basic mathematical operations (NCES, 2013a). For most, mastery does seem to come eventually; 85% of 13-year-olds and 96% of 17-year-olds demonstrated an understanding of the four basic operations in 2012. Each of these achievement levels was an increase from 1978 scores (NCES, 2013a).

In an attempt to improve mathematics education, new standards and requirements continue to be implemented. In the 1980s, many students were able to graduate from high school with only one credit in mathematics. Most states now require three or more credits in mathematics to graduate (Balfanz, McPartland, and Shaw, 2002). Students are now taking more high school math classes than ever before. The percentage of 13-year-olds enrolled in algebra doubled between 1986 and 2012, and the percentage of 17-year-olds enrolled in pre-calculus or calculus more than tripled between 1978 and 2012 (NCES, 2013a). Research has shown that elementary and middle school students need a curriculum that focuses on key topics crucial to success in the study of mathematics at the high school level (NMAP, 2008). The Common Core State Standards (CCSS) are an attempt to address this need by creating a curriculum and sequencing that is “more focused and coherent” (p.3). Standards replaced by CCSS often addressed the same concepts year after year. CCSS introduces less standards at a time and includes less overlap than previous standards (National Governors Association Center for Best Practices and Council of Chief State School Officers, 2010). Any approach that involves re-teaching the same mathematical concepts year after year is detrimental to student success (NMAP, 2008).

Statement of the Problem

The problem is increasing standards and requirements results in an increase in the number of high school students needing remediation (Balfanz et al., 2002). Standards define what students should be able to do, but they do not specify how to support students who fall behind (National Governors Association Center for Best Practices and Council of Chief State School Officers, 2010). One of the greatest areas for remediation is integer operations. While research has been done on how to best teach integer operations, there exists a lack of research on how to best remediate high school students in the area of integer operations. In fact, there exists a lack of research regarding how to remediate students who enter high school needing extra help in all areas of mathematics (Balfanz et al., 2002).

Background Information

Test scores confirm the need for remediation at the high school level. In 2012, 34% of 13-year-olds demonstrated the ability to work with signed numbers, exponents, and square roots. For 17-year-olds, 60% were able to work with signed numbers, exponents, and square roots (NCES, 2013a). Students' lack of preparation for high school math classes means that classroom time that should be devoted to covering high school standards must be spent reviewing elementary and middle school topics. In a nation-wide examination of high school math classes, it was found that on average 10-17% of the material taught in Algebra 1 classes and 11-14% of the material taught in Geometry classes belonged at the elementary and middle school level (NCES, 2013b).

High school students need the most remediation in the areas of operations with rational numbers (fractions) and operations with integers because these elementary and middle school topics are vital to student success at the high school level (Balfanz et al., 2002). Effective remediation will result in higher student achievement which can only benefit students, schools, and the nation as a whole. Research has shown that the United States could increase its GDP by improving the mathematical skills of its population (Peterson et al., 2011). A 2010 Report to the President notes that the success of the country's wealth and welfare relies on the skills of its population. The same report notes not only a lack of mathematical proficiency among U.S. students but a lack of interest in Science, Technology, Engineering, and Mathematics (STEM) related fields. Even the highest-achieving students are not gravitating towards careers in STEM areas (Executive Office of the President, 2010). If America wishes to remain a global leader, changes must be made.

Additionally, remediating students while still enrolled at the secondary level would aid in relieving the burden of remediating students at the university level. Students who enter college without a grasp of mathematics are forced to enroll in and pass remedial level classes before being allowed to enroll in the classes required for their program. These classes that must be paid for but do not count towards a student's graduation requirements can lead to student frustration, and having to take remedial classes can ultimately discourage a student to the point of giving up. The need for post-secondary remediation results in lowered academic standards, wasted tax money, and demoralized faculty at institutions of higher education (Bahr, 2008). Early and effective remediation should lead to an improvement in students' attitudes regarding mathematics.

Significance of the Study

Though teachers know the importance of teaching integer operations well, little to no research exists on how to effectively remediate students who did not master integer operations the first time they were taught. Finding a proper balance between time spent on remediating students and time spent on teaching new concepts to students is a struggle for many teachers. A nation-wide emphasis on high-stakes testing makes time management even more of a struggle for teachers. The presence of research regarding the effectiveness of various remediation techniques would help teachers make the most of classroom time allotted to remediation.

Purpose of the Study

The overall purpose of this study was to explore the impact of different approaches to remediating students in the area of integer operations. In particular, this study focused on two approaches to teaching integers: the number line model and rules to be memorized. The study responded to three research questions:

1. At the high school level, does the use of the number line model result in more effective remediation of integer operations than the use of rules to be memorized?
2. Is the use of the number line model more effective in the remediation of integers with students enrolled in higher-level math classes?
3. At the high school level, does the use of the number line model lead to a greater retention of integer operation skills than the use of rules to be memorized?

Chapter 2

Review of Related Literature

Negative numbers represent a "conceptual revolution" for students that is often taken for granted by adults (Whitacre et al., 2011, p. 2). When students begin school, the term "number" refers to the natural numbers. These counting numbers are soon joined with zero to form the whole numbers. The definition of a "number" is extended once again with the introduction of fractions. The CCSS identifies five separate experiences between kindergarten and high school graduation where students' understanding of what can constitute a "number" must change (National Governors Association Center for Best Practices and Council of Chief State School Officers, 2010). The addition of negative numbers to the number system represents more than just another broadening of the number system; it represents a "(re)defining [of] the four basic operations" (Bellamy, 2015, p. 1). For example, the minus sign that previously only meant subtraction can now take on at least three different meanings (Stephan and Akyuz, 2012).

By redefining both what is considered a number and the meaning of basic mathematical operations, students realize, perhaps for the first time in their educational careers, that some of what they have previously been taught may not always hold true (Bellamy, 2015; Whitacre et al., 2011). These overgeneralizations can result from students being taught incorrectly by well-intentioned teachers or from students using their own intuition to build on prior knowledge (Bellamy, 2015; Wessman-Enzinger and Mooney, 2014). In a journal article aimed at elementary school teachers, Karp, Bush, and Dougherty (2014) warn: "Overgeneralizing commonly accepted strategies, using imprecise vocabulary, and relying on tips and tricks that do not promote conceptual mathematical understanding can lead to misunderstanding later in students' math careers" (p. 18). Karp et al. highlight thirteen rules that appear to be true but later turn out to be true only under certain circumstances. When these rules expire, students are often left confused, furthering the view of math as a mysterious set of tricks to memorize instead of a series of interrelated concepts. Four of the thirteen rules in the article expire when students are introduced to the concept of negative numbers.

The abstract nature of negative numbers also proves to be a challenge for students; however, students are not the only ones who have been confounded by negative numbers. Historically, negative numbers were considered by many mathematicians to be absurd (Stephan and Akyuz, 2012). The challenges experienced by children coming to grips with the notion of negative numbers parallel the struggles of mathematicians to do the same. While some mathematicians were quick to embrace negative numbers due to their usefulness, others were resistant to admit that negatives were possible due to their lack of concreteness (Whitacre et al., 2011). The introduction of negative numbers represents the first time students are required to deal with numbers that cannot be represented by physical objects (Bellamy, 2015). However, manipulatives have been designed in an attempt to make negative numbers seem less abstract (Dirks, 1984; Chang, 1985; Nool, 2012; Vig, Murray, and Star 2014; Charalambous, Hill, and Mitchell, 2012).

Despite being a stumbling block for both mathematicians and secondary students, it has been suggested that negative numbers could be taught at the elementary school level. The National Council of Teachers of Mathematics (NCTM) (2000) recommends that students in third grade through fifth grade should extend the number line to explore negative numbers. Despite NCTM’s recommendation, most state standards introduce the concept of integers at the middle school level. The CCSS introduces the concept of negative numbers in the sixth grade and expects students to begin performing operations with negative numbers in the seventh grade

(National Governors Association Center for Best Practices and Council of Chief State School Officers, 2010). Therefore, high school standards are written assuming students are procedurally fluent with integers. Procedural fluency is one of five strands of mathematical proficiency identified by the National Research Council (2001). The other four strands include adaptive reasoning, strategic competence, conceptual understanding, and productive disposition. These five strands are interconnected and must all work together. Traditionally, procedural fluency has played the dominant role in the mathematics classroom for determining if a student is mathematically proficient (Suh, 2007). Fluency with integer operations is important because "students who expend too much of their available cognitive resources solving basic facts may have insufficient cognitive resources to learn and apply skills associated with math reasoning and problem solving tasks" (as cited in Poncy, Skinner, and Axtell, 2010, p. 342)

The challenges that come with teaching integers have led to the development of many different methods of teaching integer operations. Beswick (2011) divides these methods into three categories: approaches analogous to number lines, approaches based on discrete objects that cancel each other out, and approaches that extend the mathematical structure of operations with positive numbers to negative numbers. Stephan and Akyuz (2012) consider only two categories which correspond with the first two categories found by Beswick: number line models and neutralization models. Peterson (1972) embraces Beswick's third category, but he chooses to refer to it simply as "Patterns." Wessman-Enzinger and Mooney (2014) note that another common approach to teaching integer operations is to teach an algorithm, or rule, to be memorized.

To make the teaching of integers more visual, many teachers and textbooks use a number line to illustrate problems involving positive and negative numbers (Dirks, 1984). The CCSS suggests the following contexts for introducing negative numbers: “temperature above/below zero, elevation above/below sea level, credits/debits, [and] positive/negative electric charge” (National Governors Association Center for Best Practices and Council of Chief State School Officers, 2010, p. 43). In a study of textbooks, Whitacre et al. (2011) found that 89% of textbooks used elevation or temperature to introduce adding and subtracting integers, and 61% of textbooks used forward and backward movement to introduce adding and subtracting integers. The contexts of elevation, temperature, and forward and backward movement lend themselves well to being illustrated through the use of a number line. Elevation and temperature are best modeled on a vertical number line; however, forward and backward movement is best modeled on a horizontal number line (Beswick, 2011). The CCSS recommends the use of both vertical and horizontal number lines in the classroom (National Governors Association Center for Best Practices and Council of Chief State School Officers, 2010). Researchers have found numerous benefits of using the number line model to teach integers. The use of a number line can help students distinguish between a minus sign used for subtraction and a minus sign used for denoting a negative number (Çemen, 1993). Another benefit of the number line model is that it supports the ordering of negative numbers (Stephan and Akyuz, 2012). Whitman (1992) claims that having a physical model makes “concepts easier for students to understand” and avoids “rote learning of rules” (p. 34).

In the number line model, distances to the right are represented by positive numbers. Similarly, distances to the left are represented by negative numbers. Addition is performed by moving forward on the number line, and subtraction is performed by moving backwards on the number line. Multiplication of integers with the number line model is modelled by repeated addition. Division of integers with the number line relies on students to understand the connection between multiplication and division to work backwards (Beswick, 2011). The number line model, while useful, does have limitations that must be considered. Bellamy (2015) notes that while the temperature scale is often used to introduce positive and negative numbers, it can only be used to teach adding and subtracting integers. Similarly, the number line model can be used for integer multiplication and division, but it is much less obvious and useful to students (Beswick, 2007). While a model may be helpful in working with a certain type of mathematics problem, there comes a point where the model is no longer applicable or useful. Vig et al. (2014) call that the "breaking point" (p. 74) and warn: "The mathematical concepts supported by a model can be obscured or even lost entirely in the act of using the model to arrive at an answer to computational mathematics problems" (p. 85).

The use of algorithms, or rules to be memorized, is a popular choice in many classrooms. A popular example of such a rule is "keep-change-change." These three words are often taught to students to remind them what to do when they are asked to subtract a negative number (Wessman-Enzinger and Mooney, 2014). The teaching of integer operations as a set of rules to be memorized may lead to procedural fluency, but students are shortchanged in the area of conceptual understanding. For example, Wessman-Enzinger and Mooney asked students to write stories to match various integer operation problems. Students who operated using a rules mindset often created stories that did not make logical sense. Another rule often taught to students is "two negatives make a positive" (Karp et al., 2014, p. 22). This rule is meant to help students multiply and divide integers. Many students, however, apply it to other situations such as adding and subtracting integers which often leads them to get the wrong answer (Karp et al., 2014). Wessman-Enzinger and Mooney note that use of memorized rules often fails students. Sicklick (1975) warns teachers that students should never be forced to use shortcuts. Instead, they should be placed in situations that help them see the need for shortcuts and develop them themselves.

Many teachers resort to teaching integers by rules because they lack a complete understanding of integer operations, themselves. Chang (1985) notes that many teachers struggle with teaching integer operations because they themselves were taught to solve these problems using memorized rules. The ability to apply integer rules and understand the meaning behind the rules are not one and the same. Nool (2012) finds that many of his university students who are training to be elementary school teachers still struggle with adding integers. Charalambous et al. (2012) quote one teacher as saying: “I guess part of why I struggled with teaching integers is because I learned it with a lot of rules” (p. 498-499). Charalambous et al. (in a study that specifically looked at the teaching of integer operations) investigated the impact of a teacher’s mathematical knowledge for teaching (MKT) on their skills in the classroom and found that a teacher’s MKT level influences their skills in the classroom. NMAP (2008) finds that teachers are unable to teach concepts that they fail to understand themselves. Charalambous et al. suggest that teachers with higher MKT levels use more precise language and notation in the classroom when presenting mathematical content. Karp et al. (2014) identify imprecise vocabulary as a key cause of student misunderstanding.

Summary and Conclusion of the Literature

A review of the literature shows that the number line model and memorized rules are both commonly used methods for teaching integer operations (Dirks, 1984; Beswick, 2011; Wessman-Enzinger and Mooney, 2014). Having both a conceptual understanding of integers and procedural fluency with integers is key to student success (National Research Council, 2001). Use of the number line model promises to promote conceptual understanding in a way that the use of memorized rules cannot (Whitman, 1992). The most efficient way to teach integer operations is to provide students a set of rules without any justification, but this often leads students to see integer operations as “arbitrary and mysterious” (Van de Walle, 2004, p. 459). Rules to be memorized for integer operations work equally well for adding/subtracting integers and multiplying/dividing integers. However, students often misapply a rule meant for one operation for another operation (Karp et al., 2014). The number line model, on the other hand, is more suited for demonstrating the addition and subtraction of integers. It is possible to illustrate multiplication and division of integers using the number line, but students often find it less obvious and useful (Beswick, 2011) Teaching integer operations through the number line model adds a justification to answers that a set of rules to be memorized lacks. However, implementing the number line model requires more time in a classroom that is already stretched for time (Van de Walle, 2004).

Overall Purpose of the Study

The purpose of this study is to compare two widely used methods of teaching integer operations to determine if one method provides more effective remediation of integer operations in the high school classroom. Specifically, this study will compare the number line model for teaching integer operations with a traditional set of rules to be memorized.

Specific Research Questions

This study responded to three research questions:

1. At the high school level, does the use of the number line model result in more effective remediation of integer operations than the use of rules to be memorized?
2. Is the use of the number line model more effective in the remediation of integers with students enrolled in higher-level math classes?

3. At the high school level, does the use of the number line model lead to a greater retention of integer operation skills than the use of rules to be memorized?

Terminology

Common Core State Standards (CCSS) – A set of math and English standards adopted by 42 states and the District of Columbia.

Integers – Numbers with no fractional part. The integers consist of the natural numbers, zero, and the negative of the natural numbers.

Number Line Model – a horizontal or vertical line with numbers marked at specific intervals that can be used to illustrate mathematical operations.

Chapter 3

Methods

Setting

This study took place in a rural, public high school in the south-central United States. During the 2013-2014 school year, there were 180 ninth through twelfth grade students enrolled in the school. Of these students, 64% were Caucasian, 4% were Black, 1% were Asian, 1% were Hispanic, and 31% were Native American. Sixty-four percent of the students in the school and 79% of the students in the district qualified for Free/Reduced Lunch. Approximately 22% of the students were enrolled in special education services. Of the students who graduated from the school between 2010 and 2012, 31.2% of them went on to attend college. Approximately 51% of the students who went on to attend college had to take at least one remedial course (Office of Educational Quality and Accountability, 2014). This information was the latest available at the time this study was completed.

Participants

This study consisted of 72 students who were enrolled in the teacher-researcher's Algebra 1 and Algebra 2 classes during the 2015-2016 school year. Forty-nine of the students were enrolled in Algebra 1, and 23 of the students were enrolled in Algebra 2. Special education students were included in each treatment group. These students received the necessary accommodations as specified in their IEPs. All students were required to submit a signed permission slip from a parent/guardian in order to participate in the study (See Appendix A).

Only one teacher participated in this study. The teacher was also the researcher behind the study. She has four years teaching experience. She has a bachelor's degree in Mathematics and Secondary Education from the University of Tulsa. She is currently working on her master's degree in Curriculum and Instruction from the University of Texas at Arlington.

Research Design

All students were given a timed pre-test of 50 basic one-step integer operation questions (See Appendix B). Students had five minutes to complete as many problems as possible. Based on pre-test rankings, students were placed in pairs so that each student in the pair had a similar score. The first student in each pair was randomly assigned to either the control group or the treatment group. The second student in each pair was placed in the opposite group. This assures that the treatment group and control group were made up of a similar mix of abilities.

Both groups received remediation in the area of integer operations. Students were given five minutes per day to work on improving their fluency and accuracy with integer operations. Each student was given a set of flash cards to use to practice and check for understanding (See Appendix C). Every Friday during the study, students were given a timed benchmark quiz. This benchmark quiz was identical to the pre-test. Each benchmark quiz was graded and handed back on the following Monday for students to correct the problems they missed or ran out of time to complete. After three weeks, students took two timed post-tests. The first post-test occurred at the time of the normally scheduled benchmark quiz, and it was identical to the pre-test (See Appendix D). The second post-test occurred on the Monday following the first post-test, and it was made up of entirely new problems (See Appendix E). The results of these two post-tests were averaged together. Five months after the post-test, an additional timed test was given to students to measure for retention (See Appendix F).

The control group was given a set of rules to reference (and eventually memorize) for adding, subtracting, multiplying, and dividing integers (See Appendix G). Each day, students worked independently through a set of flash cards using the provided algorithms for five minutes. If students asked for help during this independent work time, the teacher-researcher showed them how to use the provided rules to solve the problem. Students in the treatment group were given a laminated horizontal number line, a plastic chip (See Appendix H), and instructions and examples of how to use a number line to add, subtract, multiply, and divide integers (See Appendices I-K). Each day, students worked independently through a set of flash cards using the horizontal number line and plastic chip to find each answer. If students asked for help during this independent work time, the teacher-researcher modeled how to use the number line and to solve the problem. The flash cards for both the control group and the treatment group were identical.

Measures

The first measure of performance used in this study was the pre-test. A set of 50 integer operation problems was administered to students. Students had five minutes to correctly answer as many questions as possible. The pre-test allowed the teacher-researcher to measure each student's fluency with integer operations. The number of problems answered correctly was recorded for each student. At the end of the study, each student's performance was measured again on two separate post-tests. The first post-test was identical to the pre-test and weekly benchmark quizzes. The second post-test featured a new set of 50 integer problems that students had not previously seen. The average of the number of problems answered correctly on the two post-tests was recorded for each student. Five months after the post-tests were administered, students were given a 50 question retention-test to measure how many integer operation problems they were still able to answer correctly in 5 minutes. Student retention was measured by subtracting their post-test score from the retention-test score.

Data Collection Procedures

This research project required student data to be collected over the course of six months. To answer the first research question, pre-test scores and post-test scores had to be collected. Scores were sorted by the type of remediation each student received and compared to determine if the use of the number line model resulted in more effective remediation than the use of rules to be memorized. To answer the second research question, pre-test scores and post-test scores also needed to be collected. However, only the scores of students in the treatment group were considered. Scores were sorted by the course each student was enrolled in and compared to determine if the number line model was more effective with students enrolled in Algebra 2 than with students enrolled in Algebra 1. For the final research question, post-test scores and retention-test scores were required to be collected. Scores were supported by the type of remediation each student received and compared to determine if the use of the number line model resulted in greater retention of integer operation skills than the use of rules to be memorized.

Data Analysis

To answer the first research question, the average pre-test and post-test scores were calculated for both the treatment group and control group. A paired samples t-test was run to check that both groups showed statistically significant group between the pre-test and post-test. An independent samples t-test was run to determine if the treatment group had statistically higher growth than the control group. To answer the second research question, the average pre-test and post-test scores were calculated for both the Algebra 1 treatment group and the Algebra 2 treatment group. Additionally, the mean and standard deviation of the increase between pre-test scores and post-test scores for the control group was calculated for both Algebra 1 students and Algebra 2 students. An independent samples t-test was run to determine if the increase in scores of Algebra 2 students was significantly higher than the increase in scores of Algebra 1 students. To answer the third research question, the mean and standard deviation of each student's retention level was calculated for the treatment group and control group. Retention was measured by finding the difference between the post-test score and an additional retention-test given at a later date. An independent samples t-test was run to see if the students in the treatment group had statistically improved retention levels than the students in the control group.

Chapter 4

Results

The following three questions were answered in this study:

1. At the high school level, does the use of the number line model result in more effective remediation of integer operations than the use of rules to be memorized?
2. Is the use of the number line model more effective in the remediation of integers with students enrolled in higher-level math classes?
3. At the high school level, does the use of the number line model lead to a greater retention of integer operation skills than the use of rules to be memorized?

Data Screening

The full data set was screened to check for missing data, homogeneity of variance, normal distribution, and outliers. Frequency histograms were obtained to determine whether the continuous variables were normally distributed. Individual histograms were analyzed for each variable to check for kurtosis and skewness.

Evaluation indicated the presence of missing data. This missing data is a result of a high level of absenteeism in the school where the project was carried out. Several students moved schools or changed teachers before all of the data could be recorded. In order to determine the method for replacing missing data, descriptive statistics were obtained using SPSS. The percent of data missing was under 5%. As a result, the missing values were left in place. Pairwise deletion was used when calculating values using SPSS.

Next, descriptive statistics were obtained on the intact data set in order to screen for outliers. For the fluency variables, no outliers were identified. Frequency histograms were obtained to determine whether the variables were normally distributed. Individual histograms were analyzed for the fluency variables (pre-test score, post-test score, and retention-test score) to check for kurtosis and skewness. Normality concerning skewness and kurtosis was evident among all variables. The skewness values were between -.688 and .636 for the fluency variables. The kurtosis values were between -.578 and -1.090 for the fluency variables. An independent samples t-test was performed in SPSS to aid in checking for homogeneity of variance. Levine's Test showed a significance level greater than 0.05 for all variables. As a result, students can be compared on each fluency variable.

Question 1 Results

To compare the use of the number line model to the use of memorized rules, a pre-test and two post-tests were given to all students. Figure 1 shows that the students in the treatment group had growth in mean scores by 13.92 points, and the students in the control group had growth in mean scores by 13.71 points.

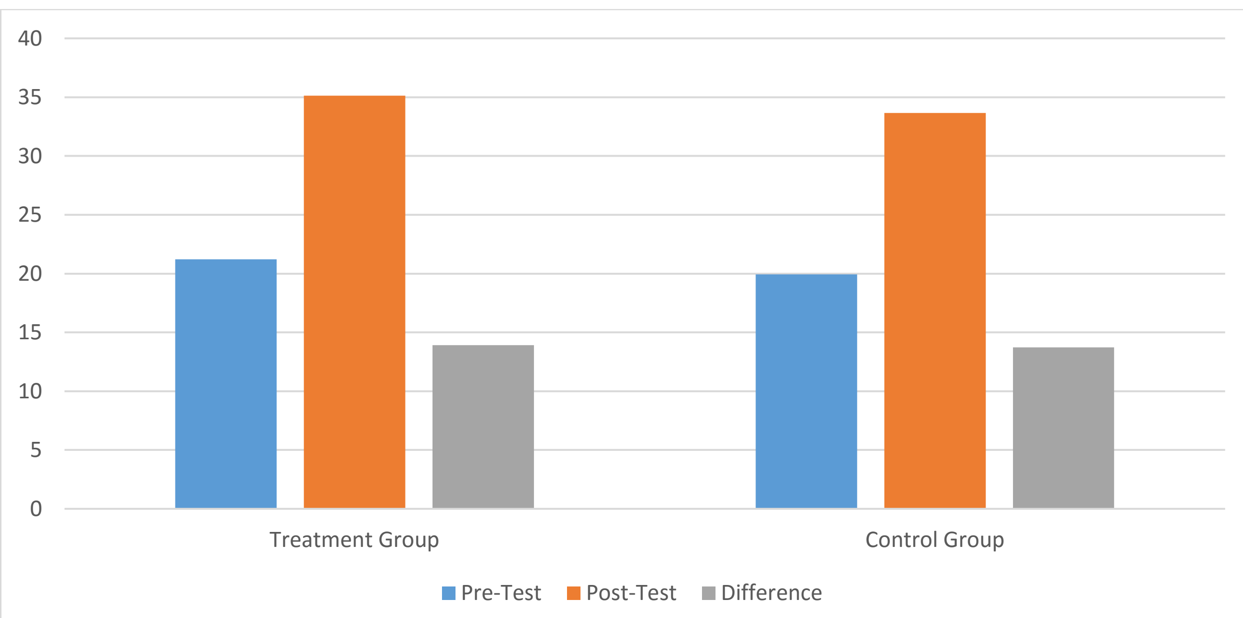

Figure 1. Pre-Test and Post-Test Means for Treatment Group and Control Group

A paired samples t-test was run to test for statistically significant growth between the methods. Table 1 shows that the treatment group showed statistically significant growth at the

.05 level, t (36) = -8.11, p<.05. The control group also showed statistically significant growth at the .05 level, t (36) = -8.03, p<.05.

Table 1

Difference Between Pre- and Post-test for Treatment and Control Groups

		Pre		Post				
Groups	*N*	*M*	*SD*	*M*	*SD*	*t*	*df*	*p*
Treatment	36	21.22	12.51	35.14	14.30	-8.11	35	≤.001
Control	36	19.94	12.95	33.65	15.21	-8.03	35	≤.001

Note. Maximum score = 50 *p<.001

To see if the treatment group had statistically higher growth in post-test scores than the control group, an independent samples t-test was run. Table 2 shows that the treatment group had a higher mean increase in scores by 0.21 points. However, this increase in scores did not prove to be a statistically significant increase in scores compared to the control group at the .05 level, t(36) = -.086, p>.05.

Table 2

Comparison of Increases in Post-test Scores for Treatment and Control Groups

		Treatment		Control				
	N	*M*	*SD*	*M*	*SD*	*t*	*df*	*p*
Increase	36	13.92	10.30	13.71	10.24	-.086	70	.932

*p>.05

Question 2 Results

To compare the effect of the number line model on integer operation remediation between students enrolled in upper-level and lower-level math classes, a pre-test and two post-tests were given to all students. Figure 2 shows that the students in the Algebra 1 treatment group had a lower pre-test average and lower post-test average than the students in the Algebra 2 treatment group. The difference between the average pre-test score and the average post-test score for the Algebra 1 treatment group was 13.61 points, while the difference for the Algebra 2 treatment group was 14.24 points.

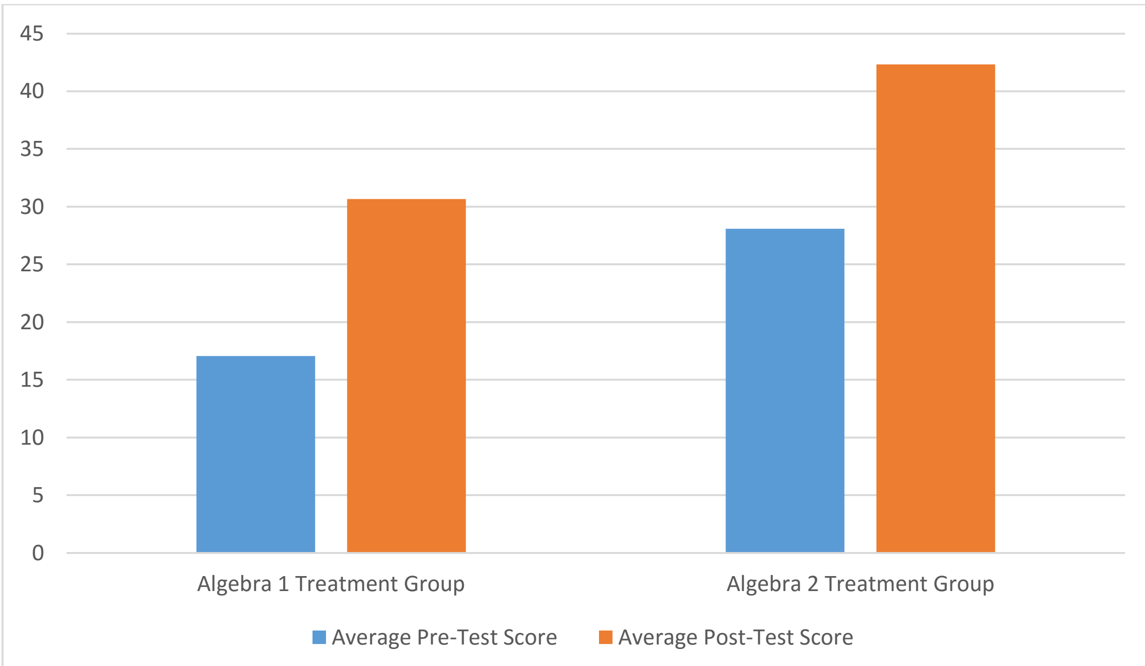

Figure 2. Pre-Test and Post-Test Means for Algebra 1 and Algebra 2 Treatment Groups

To see if students enrolled in upper-level math courses had more success with the use of the number line model in the remediation of integer operations, an independent samples t-test was run to compare the mean increases in scores between the two groups. Table 3 shows that the students enrolled in Algebra 2 had a higher mean increase in score by 0.5 points. However, this increase in scores did not prove to be a statistically significant increase in scores compared to the Algebra 1 group at the .05 level, p=.893>.05.

Table 3

Comparison of Increases in Post-Test Scores for Algebra 1 and Algebra 2 Students

	Algebra 1			Algebra 2					
Groups	*N*	*M*	*SD*	*N*	*M*	*SD*	*t*	*df*	*p*
Treatment	24	13.75	11.32	12	14.25	8.33	-.135	34	.893

*p>.05

Question 3 Results

To determine the students' retention of the remediated material, an additional test was given to students five months after the remediation occurred. Student retention was calculated by subtracting the students' post-test scores from this later retention-test. To see if the students in the treatment group retained their knowledge of integer operations better than the students in the control group, an independent sample t-test was run. Table 4 shows that the scores of the treatment group decreased an average of 1.3 points between the post-test and later test. The scores of the control group decreased an average of 3.42 points between the post-test and later test. Therefore, the treatment group demonstrated greater retention of the remediated material. However, the increased retention of the treatment group is not statistically significant compared to the control group at the .05 level, p=.333>.05.

Table 4

Comparison of Retention Levels for Experimental and Control Groups

	Experimental			Control					
	N	*M*	*SD*	*N*	*M*	*SD*	*t*	*df*	*p*
Retention	28	-1.30	7.69	30	-3.42	8.71	-.977	56	.333

Note. Retention was measured as the difference between the post-test score and an additional test given at a later date.

*p>.05

Chapter 5

Discussion

This study investigated the use of the number line model and the use of memorized rules in the remediation of integer operations at the high school level. The study showed that the use of both the number line model and the use of memorized rules led to significant student growth in the area of fluency with integer operations. However, the difference in results between the treatment group using the number line model and the control group using memorized rules was not statistically significant. The study also compared the increase in fluency for students using the number line model in Algebra 1 and Algebra 2. The study found that the difference in fluency increases between the students enrolled in different levels was not statistically significant. Finally, the study examined the retention levels of students receiving remediation using the number line model and students receiving remediation using memorized rules. The study showed that the difference in retention levels between the students using the number line model and the students using memorized rules was not statistically significant.

This study was purposed to determine if there was a statistically significant difference in the results of remediation for students who used the number line model and those who used rote memorization of rules for solving integer operation problems. According to the results of this study, the students who used the number line model did not show a significant difference in score increases between the pre-test and post-test when compared to those students who used rules to be memorized. Research has shown that the use of models boosts the conceptual understanding of students in the mathematics classroom (Whitman, 1992). The use of memorized rules often fails students (Wessman-Enzinger and Mooney, 2014) and leads them to have a view of mathematics as a disconnected set of tricks that must be memorized (Karp et al, 2014). Research, however, has noted that the use of memorized rules still efficiently improves students' procedural fluency (Van de Walle, 2004). The pre-test, post-test, and retention-test used in this study solely measured students' procedural fluency. Though the students using the number line model may have strengthened their conceptual understanding of integers, this did not lead to a statistically significant increase in their procedural fluency when compared to students who were remediated without any focus on conceptual understanding. Therefore, it can be concluded that the use of the number line model and the use of memorized rules are equally effective at improving students' procedural fluency with integer operations in a remediation setting.

The study also looked specifically at the number line model to determine if there was a statistically significant increase in scores between the students enrolled in Algebra 2 classes and the students enrolled in Algebra 1 classes. The results of the study showed that students enrolled in Algebra 2 scored higher, on average, on the post-test than students enrolled in Algebra 1. However, there was not a statistically significant difference in score increases between the students enrolled in different levels of mathematics. This finding is backed up by research which has shown that older students tend to score better than younger students when tested on the same math concepts (NCES, 2013a). Since the average increase in scores between the pre-test and post-test for Algebra 2 students was not statistically significant when compared to the average increase in scores for Algebra 1 students, it can be concluded that the number line model is equally effective for remediating students in both lower-level math classes and upper-level math classes in the area of integer operations.

Lastly, the study examined retention levels to determine if there was a statistically significant difference in the retention levels of students remediated using the number line model and students remediated using rules to be memorized. The study's results showed that students who used the number line model did not have a statistically significant increase in retention levels when compared to students who used rote memorization of integer rules. Research has shown that memorized integer rules promote procedural fluency (Van de Walle, 2004) and are often retained by students for years; this high level of retention leads many teachers to utilize them in their own classrooms (Chang, 1985; Charalambous et al., 2012). However, the National Research Council (2001) finds that mathematical proficiency requires procedural fluency, adaptive reasoning, strategic competence, and conceptual understanding. The use of models has been shown to promote conceptual understanding (Whitman, 1992) and adaptive reasoning (Vig et al., 2014). Though use of the number line model addresses more student needs than the use of memorized rules, the difference in retention levels between students remediated using the number line model and students remediated using memorized rules is not statistically significant. Therefore, it can be concluded that the number line model and the use of memorized rules are equally effective in terms of student retention.

Limitations

This study used t-tests to compare the means of various groups within the study. The difference between the means of two samples is more likely to be significant if the sample size is large. Therefore, the small sample size used was a major limitation of the study. This small sample size resulted from both the fact that the study was carried out in a small, rural public school of approximately 180 students and the fact that the participants in the study were limited to the students enrolled in the teacher-researcher's Algebra 1 and Algebra 2 classes. The teacher-researcher taught three class periods of Algebra 1 (49 students) and two class periods of Algebra 2 (23 students) during the study. Though the small sample size impacted the results of the entire study, the results for the second research question were most impacted. The second research question focused on comparing results of using the number line model between students enrolled in Algebra 1 and students enrolled in Algebra 2. Given that only half the students were assigned to use the number line model, this involved comparing the scores of 12 Algebra 2 students to the scores of 24 Algebra 1 students. The results for this second question showed that the Algebra 2 students in the treatment group had a higher mean increase in fluency than the Algebra 1 students in the treatment group, but the difference in the means was not statistically significant. The first and third research questions compared the treatment group to the control group. In each of these instances, the treatment group performed better than the treatment group, but the differences between the treatment group and the control group were not statistically significant. A larger sample size would hopefully lead to results that were statistically significant.

Another limitation of the study stems from this study being performed by a teacher-researcher. The study design called for students to be assigned to either the control group or the treatment group. To ensure the groups were comparable, the teacher-researcher gave each student a pre-test and ranked the students according to their results. Students were then placed in pairs based on their rankings. The first student in each pair was randomly assigned to either the treatment group or the control group. The second student in each pair was placed in the opposite group. This assignment method assured that the treatment group and the control group should have an even mix of students who needed minor remediation in the area of integer operations and students who needed major remediation in the area of integer operations. However, this assignment of students to remediation groups resulted in students in the same class period being assigned to receive two different types of remediation. Since the time for remediation each day was limited and different students in the same classroom were supposed to receive different forms of remediation, the teacher-researcher had to resort to providing students with written instructions for remediation. The teacher-researcher was able to demonstrate to individual students who had questions how to use either the memorized rules or number line model to solve integer operation problems. However, the teacher-researcher was unable to do a large-scale demonstration for the class since the students using the memorized rules could not be exposed to the number line model and the students using the number line model could not be exposed to the use of memorized rules. If a researcher had access to a larger school, the study could be designed so that entire classes were assigned to a single model of remediation which would allow for more effective remediation. Since the remediation provided in this study relied on the students to read a set of instructions, it is possible that some students did not thoroughly read the provided instructions and missed out on key concepts covered by the remediation materials.

Recommendations for Future Research

This research study compared the use of the number line model to the use of memorized rules in the remediation of integer operations. It would be profitable to compare these two methods to other methods for teaching integers such as the exploration of patterns (Beswick, 2011; Peterson, 1972) and the neutralization model which is often modeled using two-color counters which cancel with one another (Beswick, 2011; Stephan and Akyuz, 2012; Vig et al., 2014; Charalambous et al., 2012). Each of these models has been shown by research to be effective in the initial teaching of integer operations. However, little research exists to compare the effectiveness of methods of remediating in the area of integer operations. Additionally, further research should be conducted to study whether integer remediation is most effective when students are taught with a single method or when students are taught various methods and allowed to choose which method they would like to use to solve a certain problem. This study assigned each student to a single type of remediation, but many students strayed from this single way of looking at integers to include methods they had previously been introduced to. For example, many students assigned to use the number line model only used the number line model to solve problems they could not quickly solve in other ways.

This study looked at the short-term impact of each remediation method on students' fluency with integer operations and how well students retained their ability to operate on integers five months following the remediation period. Further research could be done to explore the impact of different teaching methods on longer-term retention. Methods of teaching and remediating integer operations and the retention that results from these methods should be compared over the course of several years. Since the CCSS places the teaching of integer operations in the seventh grade, long term retention is necessary if students are to be able to apply their knowledge of integers to the math problems they will face throughout high school and college (National Governors Association Center for Best Practices and Council of Chief State School Officers, 2010).

This study compared different methods of remediating students in the area of integer operations. This study relied on using methods which were shown to be effective in the initial instruction of integer operations and applied them to a remediation setting. In order to reduce the need for remediation, research should be done to compare the different methods for teaching integer operations to students who are learning about integers for the first time. If research could show teachers how to best teach integers initially, there would be less need for remediation of integer operations at the high school level. Finally, the impact of students' attitudes toward math should be explored to determine what impact they have on the effectiveness of different methods of remediating in the area of integer operations. Research should be done to determine if different populations of students should be remediated differently according to how they view the importance of mathematics.

Implications

This study compared the impact of the use of the number line model to the use of memorized rules to determine if one model resulted in more effective remediation in the area of integer operations than the other. The study found that there was no statistical difference between the increases in scores for the two methods of remediation. Therefore, if a teacher's aim is to improve his or her students' procedural fluency with integer operations, then the use of the number line model and the use of memorized rules should be equally effective. This study did not address students' conceptual understanding of integer operations, but previous research has shown that the use of the number line model increases students' conceptual understanding in a way that the use of memorized rules cannot (Whitman, 1992). Classroom teachers should determine what the goal of their remediation is before deciding which method of remediation to employ.

The impact of the number line model on students' fluency with integer operations was also compared across levels of mathematics (specifically between students enrolled in Algebra 1 and students enrolled in Algebra 2). The study found that there was no statistical difference between the increases in students' fluency in Algebra 1 and Algebra 2. Thus, it can be concluded that the number line model is equally effective with students enrolled in different levels of high school mathematics who require remediation with integer operations. This specific study involved the use of a laminated, horizontal number line and a plastic chip which students moved up and down the number line. Classroom teachers should realize that the use of physical manipulatives and visual models are equally effective in the remediation of integer operations with students enrolled in lower-level and upper-level math classes. Older students benefit just as much from being able to visually see a problem worked out as younger students.

Finally, the study compared the retention levels of students remediated using the number line model and students remediated using memorized rules. Student retention was measured by subtracting the students' post-test scores from their retention-test scores. The difference in the means of the retention of students using the number line model and the students using memorized rules was not statistically significant. Therefore, it can be concluded that the use of the number line model and the use of memorized rules are equally effective for ensuring student retention of integer operations. For the classroom teacher, this finding means that either method of remediation should produce long-lasting results.

Summary

In response to test scores which show that America is falling behind numerous other countries in the area of mathematics, various plans for improving mathematics education have been put into place over the last several decades (Peterson et al., 2011). Graduation requirements have been raised (Balfanz et al., 2002) and new standards have been written (National Governors Association Center for Best Practices and Council of Chief State School Officers, 2010). This increase in standards and requirements, however, has led to an increase in the number of students requiring remediation (Balfanz et al., 2002). One of the key areas of mathematics where students require remediation is the area of integer operations (NCES, 2013a). This study addressed the question of how to effectively meet this need for remediation in the area of integer operations at the high school level.

This study set out to compare two models commonly used to teach integer operations (the number line model and the use of memorized rules) to determine if one model was more effective than the other in the remediation of integer operations. The study found that the use of the number line model and the use of memorized rules were equally effective remediation tools and were equally effective in terms of student retention levels. Additionally, the study found that the number line model was equally effective at remediating students enrolled in lower-level and upper-level classes in the area of integer operations. The results of this study will help teachers to make more informed decisions about how to remediate their students in the area of integer operations. This remediation is necessary because fluency with integer operations is vital to student success in mathematics at the high school level (Balfanz et al., 2002). Time for remediation at the high school level is limited; thus, effective remediation must increase student fluency with integer operations and do so in a timely manner. This results of this study show teachers how two different methods compare in terms of increases in fluency and long-term retention. It also compares the impact of one method (the number line model) across different grade levels. Though this study did not find that one method of remediating integer operations was superior to the other, the study did find that teachers have options when it comes to effectively remediating students in the area of integer operations. Both of the methods in this study used to remediate students in the area of integer operations led to significant statistical growth. Both the use of the number line model and the use of memorized rules were shown by this study to improve students' fluency with integer operations and lead to long-term retention.

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Appendix A

Permission Slip for Research Project Participation

Research Project

Ms. Hagan is currently enrolled in a M.Ed. Program in Curriculum and Instruction—Math Studies Emphasis through the University of Texas at Arlington. As part of her coursework, she is required to complete an action research project in her classroom. Ms. Hagan has decided to study the impact of different methods of remediation on students in the area of integer operations (adding, subtracting, multiplying, and dividing positive and negative numbers.)

Students will be randomly assigned to one of two groups. Each group will receive a different set of resources to help them review what they have learned about positive and negative numbers and put it into practice. During the first quarter of the school year, students will be given five minutes per day to practice their integer operations to prepare for a weekly quiz on Fridays. Results will be analyzed to determine if one method of remediation is more effective.

There is no risk to your student to participate in this research study. If you choose to not allow your student to participate in the study, he/she will still be required to participate in the in-class activities related to the study. But, his/her results will not be included in the research study. **All results will be anonymous.**

If you would like your student's results to be included in the action research project, please sign and date below. If you have any questions, please contact Ms. Hagan at [redacted] or by e-mail at [redacted].

Parent/Guardian Signature ______________________________________ Date ______________________

Appendix B

Integer Operations Pre-Test

Name: Date: Hour:

Integer Operations Pre-Test

1. -2 + (-4) =
2. -3 + 9 =
3. -11 + 6 =
4. 9 + (-5) =
5. -3 – (-4) =
6. -7 – (+4) =
7. -6 – (-10) =
8. 6 – (+4) =
9. 4 – 9 =
10. 5(6) =
11. (-2)(-9) =
12. -8(6) =
13. (-2)(-7) =
14. 30 / -5 =
15. -12/6 =
16. -20 / -4 =
17. 54 / 9 =
18. -9 + (-2) =
19. -8 + 6 =
20. 5(-2) =
21. 56 / -8 =
22. -5 – (-11) =
23. -4 – (-14) =
24. 5 – 12 =
25. 11(6) =
26. -5 – (-9) =
27. 9 – (-5) =
28. (-2)(-6) =
29. -6(10) =
30. -13 + 9 =
31. 13 + (-8) =
32. -28 / 7 =
33. -27 / 9 =
34. 36 / -12 =
35. -8 + (-6) =
36. -9 + 15 =
37. -3 – (-7) =
38. 16 + (-5) =
39. -10 – (-12) =
40. (-6)(7) =
41. 4(-10) =
42. 14 – (+3) =
43. 9 – 16 =
44. -4(6) =
45. 3(-3) =
46. 45 / -9 =
47. -20 + 7 =
48. -6 – (-1) =
49. -36 / -6 =
50. -32 / 4 =

Appendix C

Example of Flash Cards for Integer Operation Remediation

Flash cards are printed double-sided. Therefore, the answer to the problems on this page are not shown.

-2 + (-4)	4
-3 + 9	2
-11 + 6	-5
9 + (-5)	30
-3 - (-4)	18
-7 - (+4)	-48

Appendix D

Integer Operations Post-Test A

Name: Date: Hour:

Integer Operations Post-Test A

1. -2 + (-4) =
2. -3 + 9 =
3. -11 + 6 =
4. 9 + (-5) =
5. -3 – (-4) =
6. -7 – (+4) =
7. -6 – (-10) =
8. 6 – (+4) =
9. 4 – 9 =
10. 5(6) =
11. (-2)(-9) =
12. -8(6) =
13. (-2)(-7) =
14. 30 / -5 =
15. -12/6 =
16. -20 / -4 =
17. 54 / 9 =
18. -9 + (-2) =
19. -8 + 6 =
20. 5(-2) =
21. 56 / -8 =
22. -5 – (-11) =
23. -4 – (-14) =
24. 5 – 12 =
25. 11(6) =
26. -5 – (-9) =
27. 9 – (-5) =
28. (-2)(-6) =
29. -6(10) =
30. -13 + 9 =
31. 13 + (-8) =
32. -28 / 7 =
33. -27 / 9 =
34. 36 / -12 =
35. -8 + (-6) =
36. -9 + 15 =
37. -3 – (-7) =
38. 16 + (-5) =
39. -10 – (-12) =
40. (-6)(7) =
41. 4(-10) =
42. 14 – (+3) =
43. 9 – 16 =
44. -4(6) =
45. 3(-3) =
46. 45 / -9 =
47. -20 + 7 =
48. -6 – (-1) =
49. -36 / -6 =
50. -32 / 4 =

Appendix E

Integer Operations Post-Test B

Name: Date: Hour:

Integer Operations Post-Test B

1. -3 + (-4) =
2. -3 + 5 =
3. -8 + 2 =
4. 4 + (-6) =
5. -2 – (-5) =
6. -7 – (+6) =
7. -2 – (-5) =
8. 8 – (+1) =
9. 3 – 9 =
10. 2(6) =
11. (-4)(-5) =
12. -7(6) =
13. (-3)(-7) =
14. 40 / -5 =
15. -12/4 =
16. -36 /-4 =
17. 45 / 9 =
18. -5 + (-2) =
19. -3 + 6 =
20. 3(-2) =
21. 64 / -8 =
22. -4 – (-8) =
23. -2 – (-5) =
24. 3 – 12 =
25. 7(7) =
26. -2 – (-9) =
27. 9 – (-3) =
28. (-2)(-9) =
29. -4(10) =
30. -11 + 9 =
31. 12 + (-8) =
32. -35 / 7 =
33. -45 / 9 =
34. 36 / -3 =
35. -8 + (-9) =
36. -7 + 15 =
37. -2 – (-4) =
38. 11 + (-5) =
39. -10 – (-15) =
40. (-2)(7) =
41. 3(-10) =
42. 11 – (+3) =
43. 9 – 13 =
44. -3(6) =
45. 3(-5) =
46. 54 / -9 =
47. -18 + 7 =
48. -5 – (-1) =
49. -42 / -6 =
50. -36 / 4 =

Appendix F

Integer Operations Retention-Test

Name: Date: Hour:

Integer Operations Retention-Test

1. -2 + (-4) =
2. -3 + 9 =
3. -11 + 6 =
4. 9 + (-5) =
5. -3 – (-4) =
6. -7 – (+4) =
7. -6 – (-10) =
8. 6 – (+4) =
9. 4 – 9 =
10. 5(6) =
11. (-2)(-9) =
12. -8(6) =
13. (-2)(-7) =
14. 30 / -5 =
15. -12/6 =
16. -20 / -4 =
17. 54 / 9 =
18. -9 + (-2) =
19. -8 + 6 =
20. 5(-2) =
21. 56 / -8 =
22. -5 – (-11) =
23. -4 – (-14) =
24. 5 – 12 =
25. 11(6) =
26. -5 – (-9) =
27. 9 – (-5) =
28. (-2)(-6) =
29. -6(10) =
30. -13 + 9 =
31. 13 + (-8) =
32. -28 / 7 =
33. -27 / 9 =
34. 36 / -12 =
35. -8 + (-6) =
36. -9 + 15 =
37. -3 – (-7) =
38. 16 + (-5) =
39. -10 – (-12) =
40. (-6)(7) =
41. 4(-10) =
42. 14 – (+3) =
43. 9 – 16 =
44. -4(6) =
45. 3(-3) =
46. 45 / -9 =
47. -20 + 7 =
48. -6 – (-1) =
49. -36 / -6 =
50. -32 / 4 =

Appendix G

Remediation Provided to Control Group

Integer Operations

Adding Integers

SAME SIGNS

Add and keep the sign.

Positive + Positive = Positive

Negative + Negative = Negative

DIFFERENT SIGNS

Subtract and keep the sign of the bigger number.

Subtracting Integers

Do not subtract integers.

Add the opposite!

KEEP—CHANGE—CHANGE

KEEP the sign of the first number.

CHANGE the subtraction sign to addition.

CHANGE the sign of the second number to the opposite sign.

Use the rules for adding integers.

Multiplying Integers

SAME SIGNS = Positive

DIFFERENT SIGNS = Negative

Dividing Integers

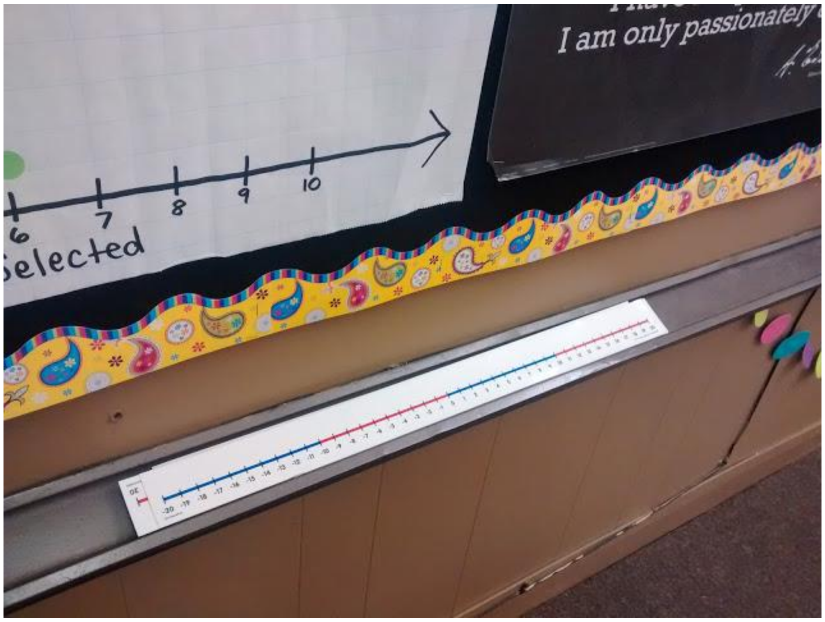
SAME SIGNS = Positive

DIFFERENT SIGNS = Negative

Appendix H

Additional Resources Provided to Treatment Group

Laminated Horizontal Number Lines



Plastic Chip with Arrow

Appendix I

Remediation for Adding/Subtracting Integers Provided to Treatment Group

Adding/Subtracting on the Number Line

ADDITION PROBLEMS

Start at zero. Face in the positive direction.

- If the first number is positive, walk forward.
- If the first number is negative, walk backwards.

To indicate addition, continue facing in the positive direction.

- If the second number is positive, walk forward.
- If the second number is negative, walk backwards.

SUBTRACTION PROBLEMS

Start at zero. Face in the positive direction.

- If the first number is positive, walk forward.
- If the first number is negative, walk backwards.

To indicate subtraction, turn around to face the opposite direction.

- If the second number is positive, walk forward.
- If the second number is negative, walk backwards.

Appendix J

Remediation for Multiplying Integers Provided to Treatment Group

Multiplying on the Number Line

Start at zero.

- If the first number is positive, face in the positive direction.
- If the first number is negative, face in the negative direction.

Determine whether you will walk forwards or back-wards.

- If the second number is positive, walk forward.
- If the second number is negative, walk backwards.

The first number tells you how many steps to take. The second number tells you the length of each step.

Appendix K

Remediation for Dividing Integers Provided to Treatment Group

Dividing on the Number Line

Start at the first number. Face towards zero.

- If the second number is positive, continue facing towards zero.
- If the second number is negative, turn around to face the opposite direction.

The second number tells you what size step you will be taking. Determine how many steps you must take to reach zero.

- If you were walking forward, your answer is positive.
- If you were walking backwards, your answer is negative.

Biographical Data

Name:	Sarah Elizabeth (Hagan) Carter
Date and Place of Birth:	November 25, 1989 Tulsa, Oklahoma
High School:	Coweta High School Coweta, Oklahoma Graduated 2008
Undergraduate College:	University of Tulsa Tulsa, Oklahoma Bachelor of Science, Mathematics and Secondary Education Graduated 2012
Teaching Experience:	High School Math Teacher Drumright Public Schools Drumright, Oklahoma 2012-Present
Graduate College:	University of Texas Arlington Arlington, Texas Master of Education, Curriculum and Instruction Expected Graduation: May 2016

Academic Honor / Honesty Statement

I have read and understand the UTA Academic Honesty clause as follows. "Academic dishonesty is a completely unacceptable mode of conduct and will not be tolerated in any form at The University of Texas at Arlington. All persons involved in academic dishonesty will be disciplined in accordance with University regulations and procedures. Discipline may include suspension or expulsion from the University. 'Academic dishonesty includes, but is not limited to, cheating, plagiarism, collusion, the submission for credit of any work or materials that are attributable in whole or in part to another person, taking an examination for another person, any act designed to give unfair advantage to a student or the attempt to commit such acts.' (Regents' Rules and Regulations, Part One, Chapter VI, Section 3, Subsection 3.2., Subdivision 3.22)."